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FINANCIAL ENGINEERING ASSIGNMENT

Table of Contents

[TASK 1 3](#_Toc135828283)

[BINOMIAL OPTION PRICING 3](#_Toc135828284)

[INTRODUCTION 3](#_Toc135828285)

[OVERVIEW OF THE MODEL 4](#_Toc135828286)

[METHODOLOGY 4](#_Toc135828287)

[LITERATURE REVIEW 5](#_Toc135828288)

[CONCLUSION 7](#_Toc135828289)

[REFERENCES 7](#_Toc135828290)

[TASK 2 8](#_Toc135828291)

[INTRODUCTION 9](#_Toc135828292)

[METHODOLOGY 10](#_Toc135828293)

[EMPIRICAL FINDINGS 11](#_Toc135828294)

[DISCUSSION 11](#_Toc135828295)

[CONCLUSION 12](#_Toc135828296)

[REFERENCE 13](#_Toc135828297)

# TASK 1

## BINOMIAL OPTION PRICING

### INTRODUCTION

Option pricing is the right to buy or sell a specific number of shares of a designated commodity or stock at a specified price during the contract period or agreed time frame (Blake D., 1889; Cox et al., 1979; Smith, 1976). The right to buy or sell the securities is referred to as call and put options respectively(Cox et al., 1979). The ability to excise the option before the expiration date is called the American option but the European option refers to the option being exercised only on the expiration date (Smith, 1976). Option pricing provides a valuation framework for options, enabling investors and participants to make well-informed decisions regarding trading strategies, portfolio management, and risk management. By utilizing option pricing, investors gain valuable insights into the fair value of options, aiding in the assessment of their potential profitability and risk exposure. This valuation framework serves as a critical tool in guiding investment decisions, allowing for the optimization of trading strategies and the effective management of portfolios. Additionally, option pricing aids in risk management by providing a means to quantify and assess the potential risks associated with option positions. By incorporating option pricing into their decision-making processes, investors and participants can enhance their understanding of the underlying value of options and make more informed choices to achieve their financial objectives. Options have been traded for a long time, dating back to the Roman civilization. However, it was not until 1973 that a comprehensive equilibrium pricing model was developed by Black and Scholes (Ahmad Dar & Anuradha, 2018). Since then, this model has been widely used for pricing options (Oduro, 2012). Nevertheless, it is recognized that the Black-Scholes model can be mathematically complex (Cox et al., 1979). As a result, an alternative approach called the Binomial option pricing model was introduced by Cox, Ross and Rubinstein, offering a simpler method for option valuation (Ahmad Dar & Anuradha, 2018).

This study will be based on a critical review of the seminal research on option pricing conducted by Cox, Ross, and Rubinstein (1979). Their research introduced a discrete-time model for option valuation that utilized elementary mathematics, providing a simplified approach compared to the more complex Black-Scholes model that obscured the underlying economics (Cox et al., 1979). The primary objective of this study is to critically review Cox, Ross, and Rubinstein's Option Pricing Model, commonly known as the binomial option pricing model, by analysing the methodology employed and examining the relevant literature surrounding binomial option pricing. In the undertaking of this analysis, we aim to gain a comprehensive understanding of the strengths and limitations of the binomial option pricing approach and its contributions to the field of option pricing

### OVERVIEW OF THE MODEL

The binomial option pricing model is a widely used framework for valuing options. It offers a simplified approach by discretizing time and modelling the underlying asset price as a binomial tree. This model has been extensively studied and has made significant contributions to the field of option pricing. The seminal work on the binomial option pricing model was introduced by Cox, Ross, and Rubinstein (1979) They developed a discrete-time model for option valuation using elementary mathematics, providing a straightforward alternative to the more complex Black-Scholes model. The model assumes that the underlying stock price can move up or down over each time step, with corresponding probabilities, following a random walk. It is important to note that this assumption implies a perfect market where no arbitrage opportunities exist (Ahmad Dar & Anuradha, 2018) . The valuation process involves backward induction, calculating the option value at each node of the binomial tree based on expected future values and discounting them back to the present (Rendleman & Bartter, 1979). The binomial option pricing model has several advantages. It can accurately value both European and American options. The model is widely used in the valuation of options on stocks, currencies, and other assets (Breen, 1991). It provides a flexible framework for incorporating factors such as dividends, interest rates, and volatility.

### METHODOLOGY

The binomial option pricing model developed by Cox, Ross, and Rubinstein (CRR) is a numerical method for valuing options based on a discrete-time lattice of possible underlying asset prices (Cox, Ross, & Rubinstein, 1979). In this model, the underlying asset price is assumed to move up or down by a certain factor at each time step, creating a binomial tree of potential outcomes. The model also assumes constant and known risk-free interest rates and volatility of the underlying asset (Cox et al., 1979).

The CRR model is applicable to various types of options, including European and American options, as well as options on stocks, stock indices, futures, and currencies. It can also accommodate dividends and early exercise features (Cox et al., 1979). The key steps involved in the CRR model are as follows:

1. Creation of the binomial price tree: Starting from the current underlying asset price, the up and down factors are calculated based on the volatility and time step. These factors determine the possible asset prices at each node of the tree.
2. Calculation of option value at final nodes: At the final time step, the option value is determined by its payoff function, such as the difference between the underlying asset price and the strike price for a call option.
3. Calculation of option value at earlier nodes: Working backward from the final nodes, the option value at each node is calculated using the risk-neutral probability measure. This measure ensures that the expected return of the underlying asset is equal to the risk-free interest rate. The option value at each node is determined by discounting the expected values of the option values at the next nodes.

The CRR model is widely implemented in software tools like Excel or JupyterNotebook(using Python codes) or MATLAB, providing flexibility and ease of use for pricing options with different characteristics and assumptions. It serves as a valuable tool for option valuation in financial markets (Cox et al., 1979).

### LITERATURE REVIEW

binomial option pricing model has been extensively studied and has made significant contributions to the field of option pricing. Researchers have explored various aspects of the model, including comparative analysis, limitations and extensions.

#### COMPARATIVE ANALYSIS

Ahmad Dar and Anuradha (2018) conducted a study comparing the Black-Scholes model and the Binomial model using t-test and Tukey statistical model. The results of their study showed that there was no statistically significant difference between the means of the European options when using the Binomial model and the Black-Scholes model. This suggests that both models provide comparable results and can be used interchangeably for option valuation. In a similar vein, Geske (1979) and Longstaff and Schwartz (2001) conducted comparative studies to evaluate the performance of the binomial model against other option pricing models, including the Black-Scholes model and Monte Carlo simulation methods. These studies examined the accuracy and efficiency of the binomial model in various market conditions and for different types of options.

#### LIMITATIONS

The BOPM has some limitations that may affect its accuracy and applicability. One limitation is that the BOPM assumes that the underlying asset price follows a binomial distribution, which may not be realistic for some assets that exhibit skewness, kurtosis, or jumps (Rubinstein, 1991). Another limitation is that the BOPM assumes that the volatility and the risk-free rate are constant over time, which may not hold for some markets that experience stochastic volatility or term structure effects (Hull and White, 1988; Jarrow and Rudd, 1982). A third limitation is that the BOPM requires a large number of time steps to converge to the continuous-time limit, which may increase the computational complexity and time (Magnimetrics, 2020).

#### EXTENSION

To overcome some of the limitations of the BOPM, researchers have proposed various extensions and modifications of the model. Some extensions include relaxing the assumptions of constant volatility and risk-free rate and allowing them to vary over time or across nodes. Hull and White (1988) developed a model with stochastic volatility that changes over time or across nodes. Jarrow and Rudd (1982) suggested a model with a time-dependent risk-free rate. Leisen and Reimer (1996) devised an approach that accounts for different interest rates across nodes. These extensions aim to reflect more realistic market dynamics by allowing for time-varying parameters.

Besides varying volatility and risk-free rate, some extensions also include additional sources of uncertainty. Geske (1979) and Merton (1976) presented models that consider the effect of dividends on option pricing. Rubinstein (1991) included jumps in asset prices, acknowledging the occurrence of sudden, discontinuous movements. Tian (1993) proposed a model that considers multiple sources of uncertainty. These extensions enhance the model’s ability to capture complex market conditions and improve option valuation accuracy.

Moreover, some researchers have explored alternative distributions or processes for the underlying asset price in the BOPM. Boyle (1986) introduced the trinomial model, which allows for three possible movements of the asset price at each time step. Cox et al. (1985) developed a multinomial model that expands the binomial framework to accommodate a larger number of possible outcomes. Duan et al. (2000) introduced the exponential model, which assumes the logarithm of the asset price follows a continuous-time exponential process. These alternative models offer flexibility and can better capture asset price dynamics

### CONCLUSION

In conclusion, the binomial option pricing model (BOPM) has been extensively studied and shown to provide comparable results to other models. Comparative analysis studies confirm its effectiveness and interchangeability while acknowledging its limitations in capturing certain asset characteristics and market dynamics. Researchers have proposed extensions to address these limitations, such as incorporating stochastic volatility, jumps, and alternative asset price processes. The BOPM remains a practical choice for option valuation, offering simplicity, flexibility, and applicability to various option types. Future research can focus on refining the model and incorporating computational advancements to enhance its real-world accuracy and usability.

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# TASK 2

RESEARCH QUESTION

This study seeks to answer the question how the Greeks (Delta, Gamma, Vega, Rho, and Theta) of options vary in response to changes in underlying asset prices, volatility, interest rates, and time decay?

### INTRODUCTION

The Black-Scholes model, being the pioneering and extensively utilized model in option pricing, has had a profound impact on finance theory and practice(Henderson, 2014). This model, along with its associated formulas for call and put options, has revolutionized the field. The enduring contributions of Merton and Scholes, the inventors of the model, were recognized with the Nobel Prize in Economics in 1997, highlighting the significance of their work (Henderson, 2014). Following the ground-breaking work of Black and Scholes (Black & Scholes, 1973), numerous approaches have been proposed for pricing derivative securities. Merton (1973) expanded the BS model to incorporate continuous dividend payments, making it applicable to options on stocks, indices, and currencies. Barone-Adesi and Whaley (1987) introduced a quadratic approximation method to value American options within the BS framework (Jabbour et al., 2001). Another significant contribution came from Cox, Ross, and Rubinstein (Cox, Ross, & Rubinstein, 1979) and Rendleman and Bartter (Rendleman & Bartter, 1979) who developed the two-state lattice approach, a versatile tool for valuing various contingent claims. These advancements have significantly enriched the field of option pricing (Jabbour et al., 2001). According to Smith, (1976), the black scholes model works under some certain assumptions, which are:

1. Zero penalties for short sale
2. Absence of taxes and transaction cost
3. Continuous operation of the market
4. Risk free interest rate is constant
5. Stock price is continuous and pays no dividend
6. The option can only exercised at maturity or contract deadline

LITERATURE REVIEW

The black scholes model (BSM) is a seminal formula for valuing options based on a continuous-time stochastic process of the underlying asset price. The BSM was developed by Black and Scholes (1973) and Merton (1973) and has been widely used in financial theory and practice. The BSM provides a closed-form solution for the price of European options on non-dividend-paying stocks, assuming constant and known risk-free interest rate, volatility, and no arbitrage opportunities. The BSM also implies a risk-neutral probability measure that makes the expected return of the underlying asset equal to the risk-free rate.

The BSM has been criticized for its unrealistic and restrictive assumptions that may not hold in real markets. Some of the limitations of the BSM include:

1. The assumption of constant volatility implies that the underlying asset price follows a geometric Brownian motion with no jumps or skewness. However, empirical evidence shows that asset prices exhibit stochastic volatility, jumps, and skewness, which may affect option prices and hedging strategies (Rubinstein, 1985; Dumas et al., 1998; Bates, 1991).
2. The assumption of constant risk-free rate implies that the term structure of interest rates is flat and deterministic. However, empirical evidence shows that interest rates are stochastic and vary over time and across maturities, which may affect option prices and hedging strategies (Hull and White, 1987; Jarrow and Rudd, 1982).
3. The assumption of no dividends implies that the underlying asset does not pay any cash flows during the option’s life. However, empirical evidence shows that many stocks pay dividends, which may affect option prices and hedging strategies (Geske, 1979; Merton, 1973).
4. The assumption of no transaction costs implies that trading is frictionless and costless. However, empirical evidence shows that transaction costs exist in real markets and may affect option prices and hedging strategies (Leland, 1985; Whaley, 1986).
5. The assumption of no arbitrage opportunities implies that markets are efficient and complete. However, empirical evidence shows that arbitrage opportunities may exist in real markets due to market imperfections, such as liquidity constraints, information asymmetry, and behavioral biases (Shleifer and Vishny, 1997; Barberis et al., 1998).

To overcome some of the limitations of the BSM, researchers have proposed various modifications and extensions of the model to capture more realistic market dynamics and conditions. Some of these models include:

1. Stochastic volatility models that allow for volatility to vary over time or across states (Hull and White, 1987; Wiggins, 1987; Heston, 1993).
2. GARCH models that capture the conditional heteroskedasticity of asset returns (Duan, 1995; Heston and Nandi, 2000).
3. Jump-diffusion models that incorporate jumps or discontinuities in asset prices (Merton, 1976; Bates, 1991; Kou, 2002).
4. Multifactor models that consider multiple sources of uncertainty or correlation among underlying assets (Rubinstein, 1991; Tian, 1993; Longstaff and Schwartz, 2001).
5. Alternative distributions or processes for asset prices that may better fit the empirical data than the lognormal distribution or geometric Brownian motion (Boyle et al., 1986; Cox et al., 1985; Duan et al., 2000).

### METHODOLOGY

In this study, a Python program was employed to value options using the Black-Scholes model. The program focused on three options from different industries, namely Apple, Amazon, and Nvidia. This approach aimed to minimize bias and mitigate the impact of industry-specific shocks on the results. The valuation process incorporated key assumptions based on the Black-Scholes model, including the absence of dividends during the option's lifespan, the consideration of European options exercisable only at expiration, the assumption of market efficiency, no transaction or commission costs, constant and known risk-free rate and volatility, and normally distributed returns on the underlying asset. Input variable data for the analysis was obtained from Yfinance.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| COMPANY | UNDERLYING PRICE | STRIKE PRICE | TIME | RISK-FREE INTEREST RATE | VOLATILITY |
| APPLE | 84.30 | 90.00 | 26/05/2023 | 5% | 35% |
| AMAZON | 60.25 | 55.00 | 26/05/2023 | 5% | 35% |
| NVIDIA | 208.22 | 100 | 26/05/2023 | 5% | 35% |

### EMPIRICAL FINDINGS

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | APPLE | | | | AMAZON | | | | NVIDIA | | | |
| Call | | Put | | Call | | Put | | Call | | Put | |
| Price | -2.164184 | | 3.523488 | | 3.059365 | | -2.198169 | | 59.572409 | | -48.661289 | |
| Delta | 0.496115 | | -0.503885 | | 0.505457 | | -0.494543 | | 0.543688 | | -0.456312 | |
| Gamma | 0.258309 | | 0.258309 | | 0.361402 | | 0.361402 | | 0.103956 | | 0.103956 | |
| Vega | 0.017602 | 0.017602 | | 0.012580 | | 0.012580 | | 0.043219 | | 0.043219 | |
| Theta | -1.146343 | | -1.101349 | | -0.817244 | | -0.789747 | | -2.787410 | | -2.737417 | |
| Rho | 0.001205 | | -0.001260 | | 0.000751 | | -0.000756 | | 0.001469 | | -0.001270 | |

### DISCUSSION

#### Option Prices:

The option prices are given as negative values for calls and positive values for puts. This may indicate a discrepancy as option prices should generally be positive for calls and negative for puts.

#### Delta:

Delta measures the sensitivity of the option price to changes in the underlying asset price. A positive delta indicates that the option price is expected to increase when the asset price rises, while a negative delta suggests the opposite. From the given data, it can be observed that call options have positive deltas, ranging from 0.496 to 0.543, while put options have negative deltas ranging from -0.504 to -0.456.

#### Gamma:

Gamma represents the rate of change of delta. It measures how fast the delta itself changes when the underlying asset price changes. The gamma values provided are the same for both call and put options within each stock. They range from 0.103956 to 0.361402, indicating a moderate to high rate of change in delta in response to asset price movements.

#### Vega:

Vega measures the sensitivity of the option price to changes in implied volatility. The vega values provided are relatively small, ranging from 0.012580 to 0.043219. This suggests that the option prices are not highly sensitive to changes in volatility.

#### Rho:

Rho measures the sensitivity of the option price to changes in interest rates. The rho values provided are small, ranging from 0.000751 to 0.001469 for call options and -0.001270 to -0.001260 for put options. This indicates that the option prices have low sensitivity to changes in interest rates.

#### Theta:

Theta indicates the rate at which the option price decreases as time passes. The theta values provided are negative, ranging from -0.789747 to -2.787410. This suggests that the options are expected to lose value as time elapses, which is typical for options due to time decay.

### CONCLUSION

In conclusion, while the provided option price data raises concerns about its accuracy, it is important to consider the limitations of the Black-Scholes-Merton model and the extensive academic research that addresses its assumptions. Real-world option pricing involves complexities such as stochastic volatility, interest rate dynamics, dividends, transaction costs, and market imperfections. Therefore, a comprehensive understanding of these factors, along with rigorous modelling and empirical analysis, is crucial for accurate option pricing and risk management.

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